DPP - Daily Practice Problems

Chapter-wise Sheets

Date : _____ **Start Time :** _____ **End Time :** _____

MATHEMATICS CM04

SYLLABUS : Principle of Mathematical Induction

Max. Marks : 120 **Marking Scheme :** (+4) for correct & (–1) for incorrect answer **Time : 60 min.**

INSTRUCTIONS : This Daily Practice Problem Sheet contains 30 MCQs. For each question only one option is correct. Darken the correct circle/ bubble in the Response Grid provided on each page.

1. Let P(n) : "$2^n < (1 \times 2 \times 3 \times ... \times n)$". Then the smallest positive integer for which P(n) is true is
(a) 1 (b) 2
(c) 3 (d) 4

2. If $P(n)$: "$46^n + 16^n + k$ is divisible by 64 for $n \in N$" is true, then the least negative integral value of k is.
(a) -1 (b) 1
(c) 2 (d) -2

3. Use principle of mathematical induction to find the value of k, where $(10^{2n-1} + 1)$ is divisible by k.
(a) 11 (b) 12
(c) 13 (d) 9

4. A student was asked to prove a statement $P(n)$ by induction. He proved that $P(k+1)$ is true whenever $P(k)$ is true for all $k > 5 \in N$ and also that $P(5)$ is true. On the basis of this he could conclude that P(n) is true
(a) for all $n \in N$
(b) for all $n > 5$
(c) for all $n \geq 5$
(d) for all $n < 5$

RESPONSE GRID	1. ⓐⓑⓒⓓ	2. ⓐⓑⓒⓓ	3. ⓐⓑⓒⓓ	4. ⓐⓑⓒⓓ

Space for Rough Work

5. Let T(k) be the statement $1+3+5+\ldots+(2k-1)=k^2+10$
Which of the following is correct?
(a) T(1) is true
(b) T(k) is true $\Rightarrow$ T($k+1$) is true
(c) T(n) is true for all $n \in \mathbf{N}$
(d) All above are correct

6. Let $S(k)=1+3+5\ldots+(2k-1)=3+k^2$. Then which of the following is true?
(a) Principle of mathematical induction can be used to prove the formula
(b) $S(k) \Rightarrow S(k+1)$
(c) $S(k) \not\Rightarrow S(k+1)$
(d) $S(1)$ is correct

7. For natural number n, $2^n(n-1)! < n^n$, if
(a) $n<2$ (b) $n>2$
(c) $n \geq 2$ (d) Never

8. For all positive integral values of n, $3^{2n}-2n+1$ is divisible by
(a) 2 (b) 4
(c) 8 (d) 12

9. For every natural number $n, n(n+1)$ is always
(a) Even (b) Odd
(c) Multiple of 3 (d) Multiple of 4

10. If $a_n=\sqrt{7+\sqrt{7+\sqrt{7+\ldots\ldots}}}$ having n radical signs then by methods of mathematical induction which is true?
(a) $a_n>7 \ \forall \ n \geq 1$ (b) $a_n<7 \ \forall \ n \geq 1$
(c) $a_n<4 \ \forall \ n \geq 1$ (d) $a_n<3 \ \forall \ n \geq 1$

11. For every positive integral value of n, $3^n > n^3$ when
(a) $n>2$ (b) $n \geq 3$
(c) $n \geq 4$ (d) $n<4$

12. If $\frac{4^n}{n+1} < \frac{(2n)!}{(n!)^2}$, then P($n$) is true for
(a) $n \geq 1$ (b) $n>0$
(c) $n<0$ (d) $n \geq 2$

13. If $n \in N$, then $x^{2n-1}+y^{2n-1}$ is divisible by
(a) $x+y$ (b) $x-y$
(c) x^2+y^2 (d) x^2+xy

14. For a positive integer n,
Let $a(n)=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots+\frac{1}{(2^n)-1}$. Then
(a) $a(100) \leq 100$ (b) $a(100)>100$
(c) $a(200) \leq 100$ (d) $a(200)<100$

RESPONSE GRID					
5. ⓐⓑⓒⓓ	**6.** ⓐⓑⓒⓓ	**7.** ⓐⓑⓒⓓ	**8.** ⓐⓑⓒⓓ	**9.** ⓐⓑⓒⓓ
10. ⓐⓑⓒⓓ	**11.** ⓐⓑⓒⓓ	**12.** ⓐⓑⓒⓓ	**13.** ⓐⓑⓒⓓ	**14.** ⓐⓑⓒⓓ

Space for Rough Work

15. $2^n > n^2$ when $n \in$ N such that

(a) $n > 2$ (b) $n > 3$

(c) $n < 5$ (d) $n \geq 5$

16. For every natural number n, $n(n^2 - 1)$ is divisible by

(a) 4 (b) 6

(c) 10 (d) None of these

17. If $49^n + 16n + \lambda$ is divisible by 64 for all $n \in$ N, then the least negative value of λ is

(a) –2 (b) –1

(c) –3 (d) – 4

18. If $n \in$ N and n is odd, then $n(n^2 - 1)$ is divisible by

(a) 24 (b) 16

(c) 32 (d) 19

19. For each $n \in N$, the correct statement is

(a) $2^n < n$ (b) $n^2 > 2n$

(c) $n^4 < 10^n$ (d) $2^{3n} > 7n + 1$

20. P(n) : $2.7^n + 3.5^n - 5$ is divisible by

(a) 24, $\forall$ n $\in$ N

(b) 21, $\forall$ n $\in$ N

(c) 35, $\forall$ n $\in$ N

(d) 50, $\forall$ n $\in$ N

21. By mathematical induction,

$\frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \ldots + \frac{1}{n(n+1)(n+2)}$ is equal to

(a) $\frac{n(n+1)}{4(n+2)(n+3)}$ (b) $\frac{n(n+3)}{4(n+1)(n+2)}$

(c) $\frac{n(n+2)}{4(n+1)(n+3)}$ (d) None of these

22. For every positive integer n, $7^n - 3^n$ is divisible by

(a) 7 (b) 3

(c) 4 (d) 5

23. For all n $\in$ N, the sum of $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15}$ is

(a) a negative integer (b) a whole number

(c) a real number (d) a natural number

24. For $n \in$ N, $x^{n+1} + (x+1)^{2n-1}$ is divisible by

(a) x (b) $x + 1$

(c) $x^2 + x + 1$ (d) $x^2 - x + 1$

25. If n is a positive integer, then $5^{2n+2} - 24n - 25$ is divisible by

(a) 574 (b) 575

(c) 674 (d) 576

26. For all n $\geq$ 1,

$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots + \frac{1}{n(n+1)} =$

(a) $\frac{n}{n+1}$ (b) $\frac{1}{n+1}$

(c) $\frac{1}{n(n+1)}$ (d) None of these

RESPONSE GRID					
15. ⓐⓑⓒⓓ	**16.** ⓐⓑⓒⓓ	**17.** ⓐⓑⓒⓓ	**18.** ⓐⓑⓒⓓ	**19.** ⓐⓑⓒⓓ
20. ⓐⓑⓒⓓ	**21.** ⓐⓑⓒⓓ	**22.** ⓐⓑⓒⓓ	**23.** ⓐⓑⓒⓓ	**24.** ⓐⓑⓒⓓ
25. ⓐⓑⓒⓓ	**26.** ⓐⓑⓒⓓ			

Space for Rough Work

27. By the principle of induction $\forall\ n \in N$, 3^{2n} when divided by 8, leaves remainder

(a) 2 (b) 3

(c) 7 (d) 1

28. **Statement-1 :** $1 + 2 + 3 + ... + n < \frac{1}{8}(2n + 1)^2$, $n \in N$.

Statement-2 : $n(n + 1)(n + 5)$ is a multiple of 3, $n \in N$.

(a) Only Statement-1 is true

(b) Only Statement-2 is true

(c) Both Statements are true

(d) Both Statements are false

29. **Statement-1 :** For every natural number $n \geq 2$,

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}$$

Statement-2 : For every natural number $n \geq 2$,

$\sqrt{n(n+1)} < n + 1$.

(a) Statement-1 is correct, Statement-2 is correct; Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is correct, Statement-2 is correct; Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is correct, Statement-2 is incorrect

(d) Statement-1 is incorrect, Statement-2 is correct.

30. For all $n \in N$, $41^n - 14^n$ is a multiple of

(a) 26 (b) 27

(c) 25 (d) None of these

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DAILY PRACTICE PROBLEM DPP CHAPTERWISE 4 - MATHEMATICS			
Total Questions	30	Total Marks	120
Attempted		Correct	
Incorrect		Net Score	
Cut-off Score	40	Qualifying Score	55
Success Gap = Net Score – Qualifying Score			
Net Score = (Correct × 4) – (Incorrect × 1)			

Space for Rough Work